Different transport regimes in a spatially-extended recirculating background

P. Castiglione[1,2], R. Festa[3] and A. Mazzino[3,4]

[1] Laboratoire de Physique Statistique, Ecole Normale Supérieure, Paris Cedex 05, France .

[2] INFM, Dipartimento di Fisica, Università "La Sapienza", I–00185 Roma, Italy.

[3] INFM, Dipartimento di Fisica, Università di Genova, I–16146 Genova, Italy.

[4] The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark .

February 5, 2008

Abstract

Passive scalar transport in a spatially-extended background of roll convection is considered in the time-periodic regime. The latter arises due to the even oscillatory instability of the cell lateral boundary, here accounted for by sinusoidal oscillations of frequency ω. By varying the latter parameter, the strength of anticorrelated regions of the velocity field can be controled and the conditions under which either an enhancement or a reduction of transport takes place can be created. Such two ubiquitous regimes are triggered by a small-scale(random) velocity field superimposed to the recirculating background. The crucial point is played by the dependence of Lagrangian trajectories on the statistical properties of the small-scale velocity field, e.g. its correlation time or its energy.

PACS 47.27Qb – Turbulent diffusion.

Transport in turbulent flows with recirculation is a problem of great interest both in the atmosphere and in the ocean [1, 2, 3]. In the atmosphere, the so-called horizontal roll vortices or, briefly, rolls are the paradigm of recirculating pattern. They can be considered as a manifestation of Bénard–Rayleigh convection occurring when conditions of combined surface heating and strong winds take place in the atmospheric boundary layer. Their depth equals the mixing layer and the ratio of lateral to vertical dimensions for a roll pair is about 3:1 (see, e.g., Ref. [4]). In the following, such type of flow configuration will be the background where particle tracers (i.e. a passive scalar) will be plunged and their statistics investigated.

An important characteristic of atmospheric rolls is their large spatial extension which makes possible the description of the related dispersion problem through an effective diffusion equation, i.e. a Fick equation for the large-scale slow-varying passive scalar concentration where the molecular diffusivity is replaced by an enhanced (eddy-) diffusivity. The existence of an asymptotic diffusive regime for the large scale concentration can be rigorously proved by using, e.g., multiscale techniques [5]. A simple Lagrangian interpretation of the asymptotic diffusive regimes is based on "central limit" argumentations. For Lagrangian chaotic flows, velocity correlations decay fastly and, as a consequence, the particle displacement, $\delta x(t)$, at the time t, is the results of the sum of almost independent advecting contributions. The result is that $\delta x(t)$ undergoes Brownian motion when observed on times larger than the typical velocity correlation time.

In this asymptotic framework, the effect on the dispersion process of the small-scale components of the velocity field is the renormalization of the effective diffusion coefficient. Notice that, as pointed out in Ref. [6], an eddy-diffusivity based description should not be possible in the presence of finite-size domains with a small number of recirculations where, necessarily, the asymptotic regime might not be reached and the dynamics is governed by transient behaviors [7]. Interesting studies on this regimes can be found, e.g., in Refs. [8, 9, 10].

For the system we are going to investigate, the characteristic length of the organized array of cells is smaller than the size of the domain. As a consequence, the eddy-diffusivity tensor, defined through the following asymptotic limit,

$$D_{\alpha\beta}^E = lim_{t\to\infty}\frac{1}{2\,t}\langle[x_\alpha(t) - x_\alpha(0)][x_\beta(t) - x_\beta(0)]\rangle, \tag{1}$$

with $x(t)$ being the particle position at the time t, and $\langle\cdot\rangle$ the average over an ensemble of tracer particles, turns out to be a well-defined mathematical quantity.

Atmospheric rolls show a wide range of regimes ranging from (almost) time independent to turbulent flow. The related dispersion phenomena are clearly strongly influenced by these different regimes and, as a consequence, transport rates vary over a wide range. An intermediate regime attracting considerable attention both theoretically [11], experimentally [12] and numerically [13] is the time-periodic regime, where the transport process is dominated by advection of tracer particles across the lateral boundary. Such regime will be the main concern of the present Letter. Specifically, the main question addressed here concerns the role of the small-scale (not explicitly resolved) components of the velocity field on the large-scale transport. We shall show that the superposition of a colored (random) noise velocity (that can be though as associated to a small-scale turbulent motion with nonvanishing memory) to the convective (deterministic) background strongly affects the transport process: either an enhanced or a reduced (with respect to the white case) eddy-diffusion may occur, depending on the frequency of the lateral roll oscillations.

The interference mechanism, recently proposed in Refs. [14, 15] for the simple, idealized parallel flow, is identified here as the responsible of this twofold behavior.

Our two-dimensional model for the roll-convection follows Ref. [12]. Specifically, the convective flow is defined by the following stream function:

$$\psi(x, y, t) = \psi_0 \sin[k_x(x + B \sin \omega t)] \sin(k_y y) \quad , \tag{2}$$

where y ranges from 0 to $L_y = 2\pi/k_y$. The stream function (2) describes single-mode, two-dimensional convection with rigid boundary condition, where the even oscillatory instability is accounted for by the term $B \sin \omega t$, representing the lateral oscillation of the roll. In Ref. [12], a quantitative comparison of the behavior in this flow with the experimental data has shown that the basic mechanisms of convective transport are well captured by the expression (2).

The periodicity of the cell along the x-axis is denoted by L ($L = 2\pi/k_x$) while its depth (along the $y-$direction) is $L/3$ (i.e. $k_y = 3k_x$). The amplitude, B, of the roll oscillations is assumed $\sim 0.13L$. The dimensionless parameter controlling the dynamics is $\epsilon \equiv \omega/\omega_R$, $\omega_R \equiv k_x k_y \psi_0$ being the characteristic frequency of particle oscillations inside the cell. The two limiting regimes $\epsilon \ll 1$ and $\epsilon \gg 1$ have been investigated analytically in Ref. [11] to obtain expressions of the eddy-diffusivity in the limit of zero molecular diffusivity. Here, we shall concentrate on the behavior for a wide range of ϵ and in the presence of diffusivity. The investigation is however not accessible by analytical techniques. In order to evaluate eddy-diffusivities, we have therefore decided to perform Monte Carlo numerical simulations of the Langevin equation

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{v}(\boldsymbol{x}(t)) + \boldsymbol{v}'(t) \quad . \tag{3}$$

The velocity field $\boldsymbol{v}(\boldsymbol{x}(t))$ is incompressible, and related to the stream-function (2) through the usual relations $v_x = \partial_y \psi$, $v_y = -\partial_x \psi$. The noise term $\boldsymbol{v}'(t)$ is a Gaussian, zero-mean random process with the colored-noise correlation function:

$$\langle v'_\alpha(t)\, v'_\beta(t') \rangle = \frac{D_0}{\tau}\, \delta_{\alpha\beta} e^{-\frac{|t-t'|}{\tau}} \quad , \tag{4}$$

where D_0 can be though as the (isotropic) eddy-diffusivity arising from the smallest (not explicitly resolved) scales of turbulent motion and τ is their correlation time. Notice that the white-in-time correlation function is obtained by taking the limit $\tau \to 0$.

From the Langevin equation (3) the expression (1) for the eddy diffusivity can be easily rewritten in terms of the Lagrangian autocorrelation $C_{\alpha\beta}(t) = \langle v_\alpha(\boldsymbol{x}(t))v_\beta(\boldsymbol{x}(0)) \rangle$:

$$D_{\alpha\beta}^E = D_0\delta_{\alpha\beta} + \int_0^\infty dt\, C_{\alpha\beta}(t). \tag{5}$$

The role played by anticorrelated regions of the velocity field (i.e. regions where $C_{\alpha\beta} < 0$ in Eq. (5)) on the large-scale transport has been investigated in Refs. [14, 15] for the class of parallel flows. Two different regimes of transport may occur depending on the extension of such regions. Specifically, when anticorrelated regions are sufficiently extended (we denote such regime with the label EAR), an increasing τ (for a fixed, small, D_0) causes a reduction of transport (with respect to $\tau = 0$) while an increasing D_0 (for a fixed, small, τ) leads to an enhancement of transport (with respect to $D_0 = 0$).

The scenario is opposite for anticorrelated regions weak enough (hereafter regime WAR), that is, D_0 leads to transport reduction while τ to transport enhancement.
We briefly recall the basic mechanisms characterizing the above two different regimes. The first mechanism works to increase the Lagrangian correlation time (and thus eddy-diffusivities): this is due to the fact that τ makes the particles of diffusing substance forget their past less rapidly than in the case $\tau = 0$. Thus, the autocorrelation function in (5) decays less rapidly than in the withe-noise case. This implies an increasing weight of regions where the velocity is strongly (positively) correlated and, as an immediate consequence, an increasing eddy-diffusivity.
The second mechanism arises for flows with closed streamlines and it is associated to the presence of anticorrelated regions of the velocity field. The correlation time, τ, is now working to increase effects of trapping due to the anticorrelated zones where the velocity is weak. This means that regions where the velocity is anticorrelated give an enhanced (again with respect to the withe-noise case) contribution to the time-integral (5). The contribution of anticorrelated regions to the time-integral being negative, a reduction of diffusion occurs.

Our main aim here is to show that in the presence of roll convection the aforesaid two mechanisms are relevant and work in competition thus governing the large-scale transport. The frequency, ω, of the lateral roll oscillation is identified here as one of the parameters controlling the crossover from the WAR and the EAR regimes. Notice that, unlike Refs. [14, 15], our control parameter, ω, is not trivially related to the extension of anticorrelated regions. The relation is intrinsic and selected by the dynamics. Indeed, by varying ω, it is possible to synchronize [16] the frequency (of order of ω_R) of particles inside the cell with the frequency, ω, of the lateral roll oscillation. Due to this synchronization mechanism, the eddy-diffusivity as a function of ω can have maxima (when oscillations are in phase) or minima (when oscillations are in phase opposition). Moreover from Eq. (5) it results that maxima (minima) of diffusion are associated to the flow configurations with the weakest (strongest) anticorrelated regions.

In order to evaluate the component of the eddy-diffusivity along the direction (e.g., the x-axis) of the lateral roll oscillation as a function of ω, numerical integration of Eq. (3) has been made by using a second-order Runge-Kutta scheme and then performing a linear fit of $\langle [x(t) - x(0)]^2 \rangle$ vs t. Averages are made over different realizations and performed
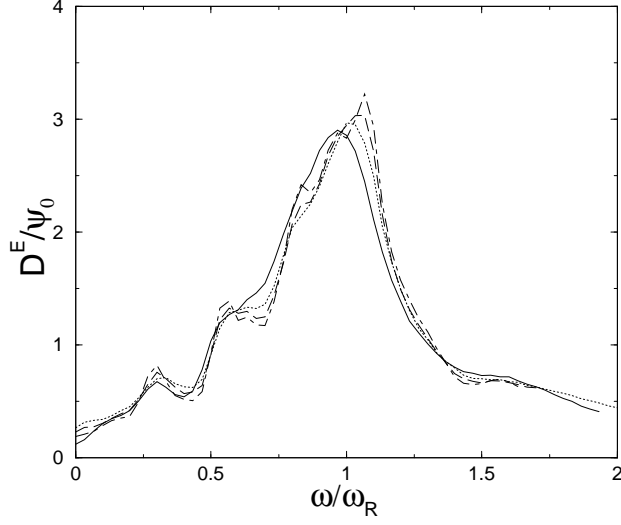
4

Figure 1: Behavior of the eddy-diffusivity (component along the direction of the lateral roll oscillations) vs the lateral roll oscillation frequency. Full line: $\tau = 0$; dotted line: $\tau/t_R = 0.24$; long-dashed line: $\tau/t_R = 0.48$; dot-dashed line: $\tau/t_R = 0.95$.

by uniformly distributing 10^6 particles in the basic periodic cell. The system evolution has been computed up to times $10^4 \, t_R$, where $t_R \equiv 2\pi/\omega_R$.

The x-component, D^E, of the eddy-diffusivity *vs* the frequency, ω, of the roll oscillation is shown in Fig. 1 for $D_0/\psi_0 = 5 \times 10^{-3}$ and different values of τ: $\tau = 0$ (full line), $\tau/t_R = 0.24$ (dotted line), $\tau/t_R = 0.48$ (long-dashed line) and $\tau/t_R = 0.95$ (dot-dashed line). A few comments are in order. Eddy-diffusivity shows maxima originated from the resonance between the lateral roll oscillation frequency and the characteristic frequencies of the particle motion. Moreover, the effect of τ on the shape of the peaks is twofold: firstly, the variation of τ causes a shifting of maxima positions and, secondly, the larger (smaller) τ, the higher (lower) the peaks. The first feature (particularly evident for ω corresponding to the highest peak) suggests that the correlation time, τ, acts to renormalize the large scale velocity. The result is that an enhanced (with respect to the white-in-time case) convective velocity governs the transport and, in order to have resonance when increasing τ, the roll oscillation frequency must thus follow the increasing velocity. The renormalizing effect of τ has been identified perturbatively in Ref. [17] for small τ. Our results seem to suggest a generalization for finite τ.

Concerning the second effect played by τ, this means that peaks and valley of the eddy-diffusivity profile are associated to regions of type WAR and EAR, respectively. In terms
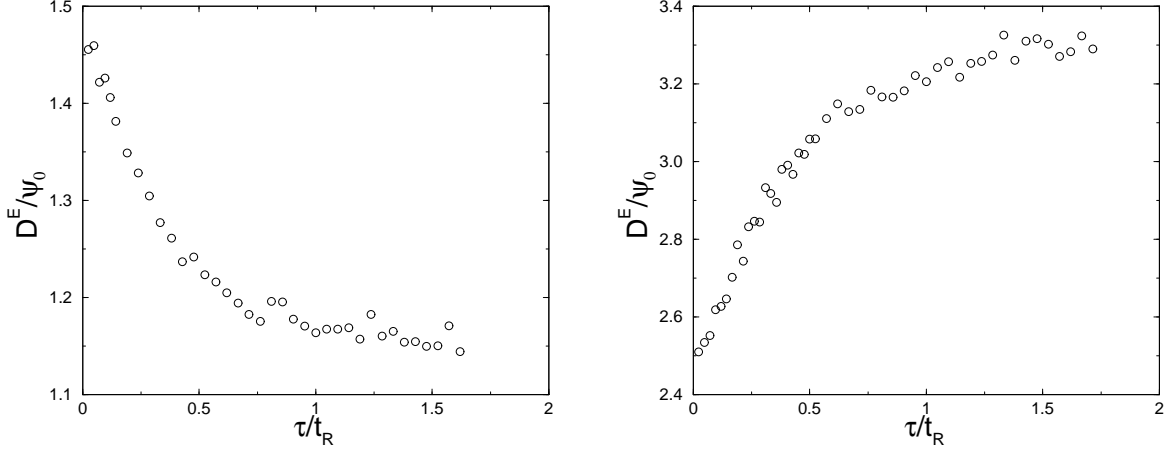
5

Figure 2: Behavior of the eddy-diffusivity vs the correlation time of the small-scale velocity $\mathbf{v}'$. On the left: $\omega/\omega_R=0.67$, corresponding to a local minimum in the eddy-diffusivity profile of Fig. 1; on the right: $\omega/\omega_R=1.1$, corresponding to the highest peak (see again Fig. 1). In both cases, $D_0/\psi_0 = 5 \times 10^{-3}$.

of the two mechanisms associated to such regions, the first is the winner for value of ω corresponding to the peaks in the eddy-diffusivity profile (where the contribution of anticorrelated regions is weak), while the second dominates for values of ω corresponding to local minima in the eddy-diffusivity profile (where the weight of anticorrelated regions is strong). This can be easily seen also from Fig. 2 where behaviors of the eddy-diffusivity as a function of τ are shown for two different value of ω: $\omega/\omega_R = 0.67$ (on the left) and $\omega/\omega_R = 1.1$ (on the right). The former value corresponds to a local minimum in the eddy-diffusivity profile, while the latter to the highest peak (see Fig. 1).

It is remarkable that the scenario above described is opposite when fixing τ and varying the (small-scale) bare eddy-diffusivity D_0. This can be easily observed from Fig. 3 where behaviors of the eddy-diffusivity are now shown as a function of D_0, keeping τ fixed and equal to zero. As in Fig.2, on the left we have $\omega/\omega_R = 0.67$ while, on the right, $\omega/\omega_R = 1.1$. The physical reason of such behavior can be easily grasped from the aforesaid mechanisms but now recalling the fact that an increasing D_0 makes the particles of diffusing substance forget their past more rapidly (rather than less rapidly as it happens by increasing τ). This effect causes a reduction in the transport. On the other hand, concerning the second mechanism, trapping due to anticorrelated regions is now less effective. Indeed, leaving the region of trapping is easier when increasing D_0. This fact leads to a reduction of the weight of the negative contribution to the time-integral (5) giving the eddy-diffusivity and, as a consequence, transport is enhanced.

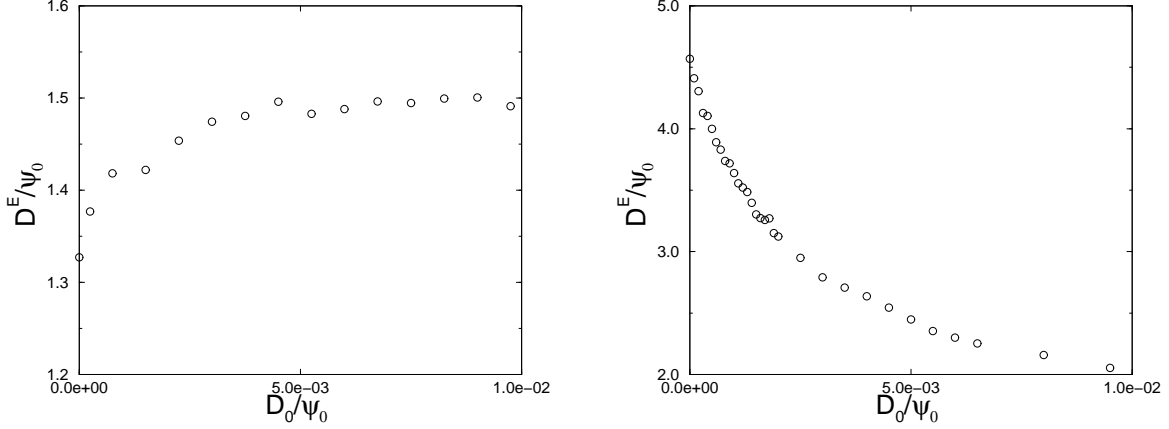The final result is thus a complete symmetry between the following operation: increasing

6

Figure 3: As in Fig. 2 but the eddy-diffusivity is plotted vs D_0 for a fixed value of the correlation time: $\tau = 0$.

τ (for a fixed D_0) and decreasing D_0 (for a fixed τ). Large-scale transport seems thus controlled by a parameter $\propto D_0/\tau$. Roughly speaking, this is easily understood if we observe that when increasing τ the particle motion along the Lagrangian trajectories becomes more and more coherent. Conversingly, coherence becomes lost when, for a fixed τ, we decrease D_0. In this sense, the dependence of Lagrangian trajectories on the statistical properties of the small scale velocity is crucial in this problem.

In conclusion, two ubiquitous different regimes of transport have been identified here as relevant in the time-periodic roll circulation. A key role to select such two regimes of transport enhancement/reduction is due to the interplay between the even oscillatory instability of the cell and the statistical properties of the small-scale (random) velocity (e.g. their correlation time or their energy). Specifically, in the model here considered, the even instability is accounted for by a sinusoidal lateral boundary oscillation with frequency ω, while small-scale velocity activity is described by a bare diffusivity, D_0, within a Gaussian, zero-mean random process with correlation time τ. When varying ω, the eddy diffusivity profile appears very structured with sharp peaks separated by evident valley. Peaks turn out to be associated to transport enhancement when increasing τ (for a fixed D_0) or, conversingly, when reducing D_0 (for a fixed τ). The situation is opposite for values of ω corresponding to minima in the eddy-diffusivity profile. The physical key role is played by synchronization mechanisms (from which the structured eddy-diffusivity profile arises) and by the strength of the anticorrelated regions of the velocity field, the weight of which in the time-integral giving the eddy-diffusivity can be either enhanced (by reducing D_0/τ) or reduced (by increasing D_0/τ), thus affecting in different ways the large-scale transport.

7

References

[1] P.B. Rhines and W.R. Young, *J. Fluid Mech.* **122**, 347, (1982).

[2] R.S. Pickart, *J. Geophys. Res.* **93**, 6761, (1988).

[3] K.J. Richards, Y. Jia and C.F. Rogers, *J. Phys. Oceanogr.* **25**, 873, (1995).

[4] R.A. Brown, *J. Atmos. Sci.* **27**, 742, (1970).

[5] A. Bensoussan, J.-L. Lions and G. Papanicolaou, *Asymptotic Analysis for Periodic Structures* (North-Holland, Amsterdam, 1978)

[6] P. Castiglione, M. Cencini, A. Vulpiani, and E. Zambianchi, "Transport in finite size systems: an exit time approach", e-Print archive chao-dyn/9903014.

[7] W.R. Young, *J. Fluid Mech.* **193**, 129, (1988).

[8] R. Sabot and M. A. Dubois, *Phys. Lett. A* **212**, 201, (1996).

[9] V. Artale, G. Boffetta, A. Celani, M. Cencini and A. Vulpiani, *Phys. Fluids* **9**, 3162, (1997).

[10] G. Károlyi and T. Tél, *Phys. Rep.* **290**, 125, (1997).

[11] A.A. Chernikov, A.I. Neishtadt, A.V. Rogal'sky and V.Z. Yakhnin, *Chaos* **1**, 206, (1991).

[12] T.H. Solomon and J.P. Gollub, *Phys Rev. A* **38**, 6280, (1988).

[13] P. Castiglione, A. Mazzino, P. Muratore-Ginanneschi and A. Vulpiani *Physica D*, **134**, 75, (1999).

[14] A. Mazzino and M. Vergassola, *Europhys. Lett.*, **37**, 535 (1997)

[15] A. Mazzino and P. Castiglione, *Europhys. Lett.*, **45**, 476 (1999)

[16] P. Castiglione, A Crisanti, A. Mazzino, M. Vergassola and A. Vulpiani, *J. Phys. A* **31**, 7197, (1998).

[17] P. Castiglione and A. Crisanti, *Phys. Rev. E*, **59**, 3926 (1999)